Solution for Problem 1:

We have two coupled differential equations. They can be solved using Euler's method. First, we set the drag constant k to zero to check the accuracy of the numerical solution. For k=0, the differential equations become $\frac{dv_x}{dt}=0$ and $\frac{dv_y}{dt}=-g$. Therefore, $v_y=v_y(0)-gt$ and $y=y(0)+v_y(0)t-gt^2/2$. For $y(0)=0$, $v_y(0)=4.15$ m/s, $g=9.81$ m/s^2, and $k=0$, we find $t=0.846$ s. We could calculate the length of the jump (neglecting air resistance) from this last equation, but this is not necessary to check the accuracy of the solution. For a step size of 0.01 s, the jump time t becomes 0.85 s and the length of the jump is 8.03 m. For a step size of 0.005 s, these values are the same. Therefore, the error of the jump time in our calculation is less then 0.005 s (the value of the step size). The error in the length of the jump is 0.05 m. This should be acceptable for our purposes.

Next, we solve the equations including air resistance. Now, the jump time is 0.83 s (not much different), but the length of the jump is only 7.77 m, much less than the actual length of the jump, which is 8.90 m. The accuracy of our calculation should be about the same with and without air resistance, since the numerical results are similar in both cases. Under normal conditions, when the air density is 1.3 kg/m^3, the length of the jump is 7.70 m, only 0.07 m less than in thin air.

The value of g changes only about 5 parts in 10000 between sea level and a location one mile above sea level. Therefore, g is at least 9.805 m/s^2 in Mexico City, if it is 9.81 m/s^2 at sea level. With this value of g, the length of the jump is still 7.77 m.

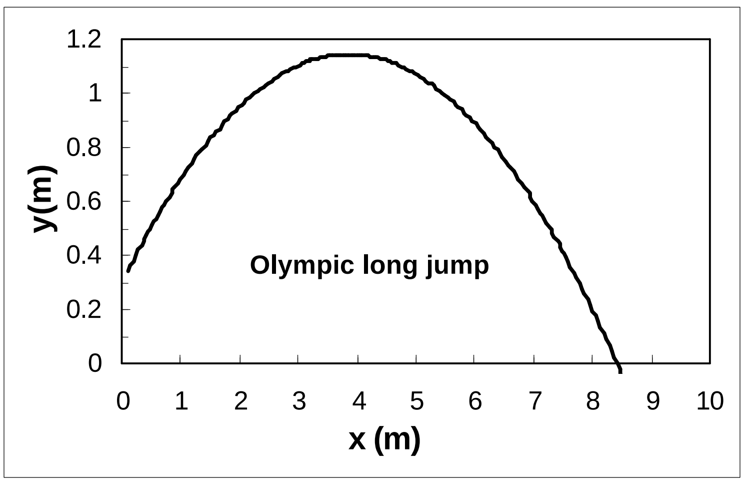

Figure 1: Range of the long jump, assuming $y(0)$=0.3 m.

Nothing we have considered so far can explain the discrepancy between the actual length of the jump and its calculated value. Since we have carefully tested the numerical accuracy of our method, we can only conclude that (i) either the starting values are different or (ii) the physical model (i.e., the differential equations) are wrong. Actually, the solution to our puzzle is similar: We have treated the athlete as a point mass. That is not a good approximation. Actually, a long jumper stretches his/her legs forward at the end of the jump. That should gain an extra two feet or so. Also, the center of mass is about three feet above the ground at the beginning of the jump and only about one or two feet above the ground at the end. This will add an extra two feet or so to the length of the jump (compare your graph). It is difficult to calculate the exact length of the jump without additional input data, but at least we have a found a possible source for the discrepancy between calculation and data.

<u>Solution for Problem 2:</u>

A mass *m* of water loses heat to its surroundings by radiation (note that we neglect convection and conduction, the other two heat transport mechanism) according the the Stefan-Boltzmann law

$$mc\frac{dT}{dt} = -4\pi R^2 \sigma (T^4 - T_1^4) \, .$$

This is an ordinary differential equation, but it is nonlinear. I decided to solve it using Euler's method in the Excel spreadsheet program. The radius *R* of the mass of water (not given in the problem) can be calculated from the water's density. We find R=6.20 cm. The surface of area of the water mass is therefore 0.048 m^2. Now we have all the input data for solving the equation. I chose a step size of 720 s (12 min), but verified that the results do not depend on the step size. The temperature difference as a function of time is given in Fig. 2. We can see that it takes about 2 hours and 20 minutes for the temperature difference to drop by 50%.

If we double the mass, we also have to consider that this will change the radius and the surface area. For a mass of m=2 kg, the radius is now R=7.82 cm and the surface area A=0.0768 m^2. The spreadsheet calculation shows that it now takes about 3 hours for the temperature difference to decrease by 50%. This result is not surprising, since doubling the mass only increases the surface area by 2 to the power of 1.5, which is 1.58. (The area increases sublinearly with mass.) Therefore, the cooling time increases by 2/1.58 or 26%.

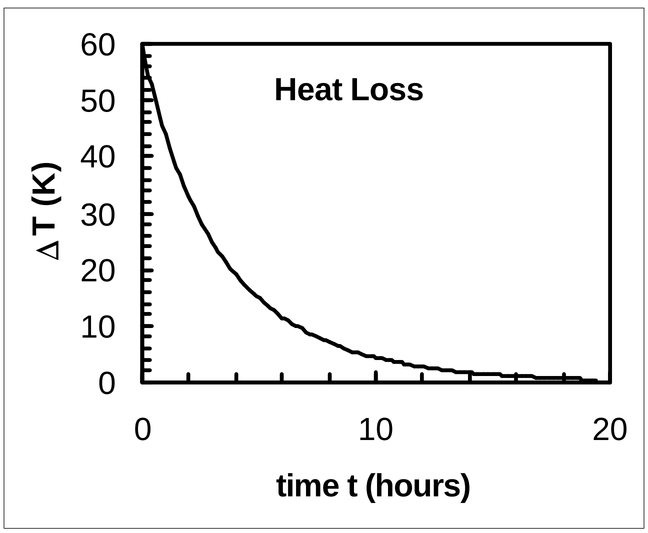

Figure 2: Temperature difference between the water bath and its surroundings as a function of time in hours.

<u>Solution for Problem 3:</u>

The ion density is governed by the ordinary nonlinear differential equation (DEQ) $\frac{dn}{dt} = A - kn^2$. Since the DEQ is nonlinear, it can only be solved numerically. However, this is not too difficult, since the coefficients *A* and *k* are constant. I used an Excel spreadsheet and Euler's method. The result is shown by the solid line in Fig. 3.

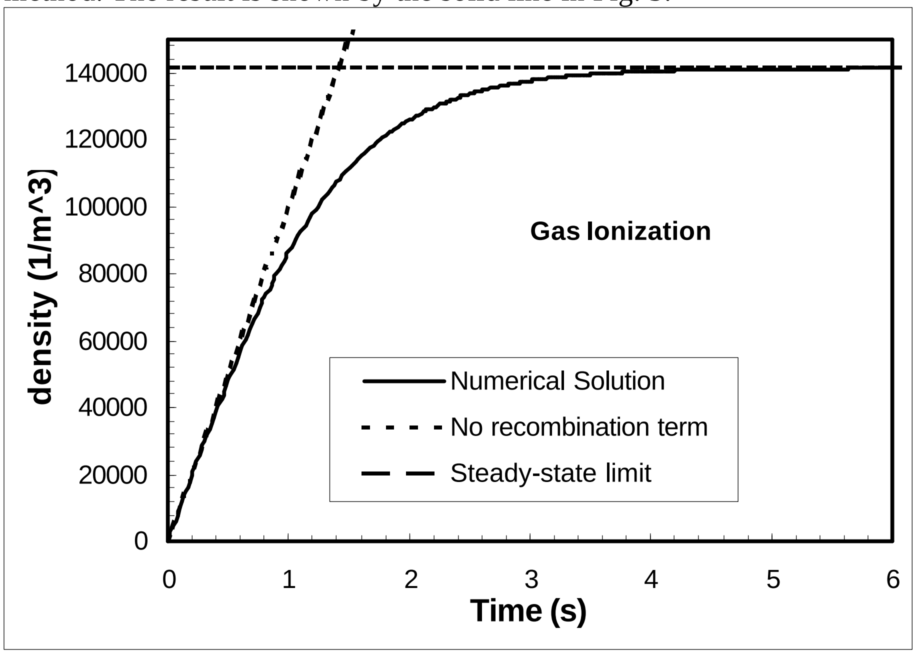

Figure 3: Ion density as a function of time.

We first discuss two limiting cases: If k=0 (no recombination), n(t)=At. In this case, the density increases linearly, see the dotted line in Fig. 3. For very long times, the solution reaches a steady-state limit, similar to the skydiver problem we looked at before. In the steady-state limit, the left hand side of the DEQ is zero, therefore $n^2=A/k$. This steady-state limit is shown by the dashed line in Fig. 3. The numerical solution to the full problem (using a step size h=0.02 s) is given by the solid line in Fig. 3. The gas reaches one half of its steady-state density after about 0.77 s. This value is independent of the step size, therefore we can assume that our numerical solution is reasonably accurate.

Changing the value of *A* affects the initial slope of the density (given by the dotted line) and also the steady-state limit. Changing the value of *k* only affects the steady-state limit. For a constant *A*/*k* ratio, the value of *A* determines how fast the density reaches its steady-state limit.